SOCIETY OF ACTUARIES/CASUALTY ACTUARIAL SOCIETY

EXAM FM FINANCIAL MATHEMATICS

EXAM FM SAMPLE QUESTIONS

Some of the questions in this study note are taken from past SOA/CAS examinations.

FM-09-05 PRINTED IN U.S.A.

These questions are representative of the types of questions that might be asked of candidates sitting for the new examination on Financial Mathematics (2/FM). These questions are intended to represent the depth of understanding required of candidates. The distribution of questions by topic is not intended to represent the distribution of questions on future exams.

1.

Bruce deposits 100 into a bank account. His account is credited interest at a nominal rate of interest of 4% convertible semiannually.

At the same time, Peter deposits 100 into a separate account. Peter's account is credited interest at a force of interest of δ.

After 7.25 years, the value of each account is the same.

Calculate δ.

(A) 0.0388

(B) 0.0392

(C) 0.0396

(D) 0.0404

(E) 0.0414

2.

Kathryn deposits 100 into an account at the beginning of each 4-year period for 40 years. The account credits interest at an annual effective interest rate of i.

The accumulated amount in the account at the end of 40 years is X, which is 5 times the accumulated amount in the account at the end of 20 years.

Calculate X.

(A) 4695

(B) 5070

(C) 5445

(D) 5820

(E) 6195

3.

Eric deposits 100 into a savings account at time 0, which pays interest at a nominal rate of i, compounded semiannually.

Mike deposits 200 into a different savings account at time 0, which pays simple interest at an annual rate of i.

Eric and Mike earn the same amount of interest during the last 6 months of the 8^{th} year.

Calculate i.

(A) 9.06%

(B) 9.26%

(C) 9.46%

(D) 9.66%

(E) 9.86%

4.

John borrows 10,000 for 10 years at an annual effective interest rate of 10%. He can repay this loan using the amortization method with payments of 1,627.45 at the end of each year. Instead, John repays the 10,000 using a sinking fund that pays an annual effective interest rate of 14%. The deposits to the sinking fund are equal to 1,627.45 minus the interest on the loan and are made at the end of each year for 10 years.

Determine the balance in the sinking fund immediately after repayment of the loan.

(A) 2,130

(B) 2,180

(C) 2,230

(D) 2,300

(E) 2,370

5.

An association had a fund balance of 75 on January 1 and 60 on December 31. At the end of every month during the year, the association deposited 10 from membership fees. There were withdrawals of 5 on February 28, 25 on June 30, 80 on October 15, and 35 on October 31.

Calculate the dollar-weighted (money-weighted) rate of return for the year.

(A) 9.0%

(B) 9.5%

(C) 10.0%

(D) 10.5%

(E) 11.0%

6.

A perpetuity costs 77.1 and makes annual payments at the end of the year.

The perpetuity pays 1 at the end of year 2, 2 at the end of year 3, …., n at the end of year (n+1). After year (n+1), the payments remain constant at n. The annual effective interest rate is 10.5%.

Calculate n.

(A) 17

(B) 18

(C) 19

(D) 20

(E) 21

7.

1000 is deposited into Fund X, which earns an annual effective rate of 6%. At the end of each year, the interest earned plus an additional 100 is withdrawn from the fund. At the end of the tenth year, the fund is depleted.

The annual withdrawals of interest and principal are deposited into Fund Y, which earns an annual effective rate of 9%.

Determine the accumulated value of Fund Y at the end of year 10.

(A) 1519

(B) 1819

(C) 2085

(D) 2273

(E) 2431

8.

You are given the following table of interest rates:

Calendar Year of Original Investment	Investment Year Rates (in %)					Portfolio Rates (in %)
y	i_1^y	i_2^y	i_3^y	i_4^y	i_5^y	i^{y+5}
1992	8.25	8.25	8.4	8.5	8.5	8.35
1993	8.5	8.7	8.75	8.9	9.0	8.6
1994	9.0	9.0	9.1	9.1	9.2	8.85
1995	9.0	9.1	9.2	9.3	9.4	9.1
1996	9.25	9.35	9.5	9.55	9.6	9.35
1997	9.5	9.5	9.6	9.7	9.7	
1998	10.0	10.0	9.9	9.8		
1999	10.0	9.8	9.7			
2000	9.5	9.5				
2001	9.0					

A person deposits 1000 on January 1, 1997. Let the following be the accumulated value of the 1000 on January 1, 2000:

P: under the investment year method
Q: under the portfolio yield method
R: where the balance is withdrawn at the end of every year and is reinvested at the new money rate

Determine the ranking of P, Q, and R.

(A) $P > Q > R$

(B) $P > R > Q$

(C) $Q > P > R$

(D) $R > P > Q$

(E) $R > Q > P$

9.

A 20-year loan of 1000 is repaid with payments at the end of each year.

Each of the first ten payments equals 150% of the amount of interest due. Each of the last ten payments is *X*.

The lender charges interest at an annual effective rate of 10%.

Calculate *X*.

(A) 32

(B) 57

(C) 70

(D) 97

(E) 117

10.

A 10,000 par value 10-year bond with 8% annual coupons is bought at a premium to yield an annual effective rate of 6%.

Calculate the interest portion of the 7^{th} coupon.

(A) 632

(B) 642

(C) 651

(D) 660

(E) 667

11.

A perpetuity-immediate pays 100 per year. Immediately after the fifth payment, the perpetuity is exchanged for a 25-year annuity-immediate that will pay X at the end of the first year. Each subsequent annual payment will be 8% greater than the preceding payment.

The annual effective rate of interest is 8%.

Calculate X.

(A) 54

(B) 64

(C) 74

(D) 84

(E) 94

12.

Jeff deposits 10 into a fund today and 20 fifteen years later. Interest is credited at a nominal discount rate of *d* compounded quarterly for the first 10 years, and at a nominal interest rate of 6% compounded semiannually thereafter. The accumulated balance in the fund at the end of 30 years is 100.

Calculate *d*.

(A) 4.33%

(B) 4.43%

(C) 4.53%

(D) 4.63%

(E) 4.73%

13.

Ernie makes deposits of 100 at time 0, and X at time 3. The fund grows at a force of interest

$$\delta_t = \frac{t^2}{100},\ t > 0.$$

The amount of interest earned from time 3 to time 6 is also X.

Calculate X.

(A) 385

(B) 485

(C) 585

(D) 685

(E) 785

14.

Mike buys a perpetuity-immediate with varying annual payments. During the first 5 years, the payment is constant and equal to 10. Beginning in year 6, the payments start to increase. For year 6 and all future years, the current year's payment is *K*% larger than the previous year's payment.

At an annual effective interest rate of 9.2%, the perpetuity has a present value of 167.50.

Calculate K, given K < 9.2.

(A) 4.0

(B) 4.2

(C) 4.4

(D) 4.6

(E) 4.8

15.

A 10-year loan of 2000 is to be repaid with payments at the end of each year. It can be repaid under the following two options:

(i) Equal annual payments at an annual effective rate of 8.07%.

(ii) Installments of 200 each year plus interest on the unpaid balance at an annual effective rate of i.

The sum of the payments under option (i) equals the sum of the payments under option (ii).

Determine i.

(A) 8.75%

(B) 9.00%

(C) 9.25%

(D) 9.50%

(E) 9.75%

16.

A loan is amortized over five years with monthly payments at a nominal interest rate of 9% compounded monthly. The first payment is 1000 and is to be paid one month from the date of the loan. Each succeeding monthly payment will be 2% lower than the prior payment.

Calculate the outstanding loan balance immediately after the 40^{th} payment is made.

(A) 6751

(B) 6889

(C) 6941

(D) 7030

(E) 7344

17.

To accumulate 8000 at the end of $3n$ years, deposits of 98 are made at the end of each of the first n years and 196 at the end of each of the next $2n$ years.

The annual effective rate of interest is i. You are given $(1 + i)^n = 2.0$.

Determine i.

(A) 11.25%

(B) 11.75%

(C) 12.25%

(D) 12.75%

(E) 13.25%

18.

Olga buys a 5-year increasing annuity for X.

Olga will receive 2 at the end of the first month, 4 at the end of the second month, and for each month thereafter the payment increases by 2.

The nominal interest rate is 9% convertible quarterly.

Calculate X.

(A) 2680

(B) 2730

(C) 2780

(D) 2830

(E) 2880

19.

You are given the following information about the activity in two different investment accounts:

Account K			
	Fund value	Activity	
Date	before activity	Deposit	Withdrawal
January 1, 1999	100.0		
July 1, 1999	125.0		X
October 1, 1999	110.0	$2X$	
December 31, 1999	125.0		

Account L			
	Fund value	Activity	
Date	before activity	Deposit	Withdrawal
January 1, 1999	100.0		
July 1, 1999	125.0		X
December 31, 1999	105.8		

During 1999, the dollar-weighted (money-weighted) return for investment account K equals the time-weighted return for investment account L, which equals i.

Calculate i.

(A) 10%

(B) 12%

(C) 15%

(D) 18%

(E) 20%

20.

David can receive one of the following two payment streams:

(i) 100 at time 0, 200 at time n, and 300 at time $2n$

(ii) 600 at time 10

At an annual effective interest rate of i, the present values of the two streams are equal.

Given $v^n = 0.76$, determine i.

(A) 3.5%

(B) 4.0%

(C) 4.5%

(D) 5.0%

(E) 5.5%

21.

Payments are made to an account at a continuous rate of $(8k + tk)$, where $0 \le t \le 10$.

Interest is credited at a force of interest $\delta_t = \frac{1}{8+t}$.

After 10 years, the account is worth 20,000.

Calculate k.

(A) 111

(B) 116

(C) 121

(D) 126

(E) 131

22.

You have decided to invest in Bond X, an *n*-year bond with semi-annual coupons and the following characteristics:

- Par value is 1000.
- The ratio of the semi-annual coupon rate to the desired semi-annual yield rate, $\frac{r}{i}$, is 1.03125.
- The present value of the redemption value is 381.50.

Given $v^n = 0.5889$, what is the price of bond X?

(A) 1019

(B) 1029

(C) 1050

(D) 1055

(E) 1072

23.

Project P requires an investment of 4000 at time 0. The investment pays 2000 at time 1 and 4000 at time 2.

Project Q requires an investment of X at time 2. The investment pays 2000 at time 0 and 4000 at time 1.

The net present values of the two projects are equal at an interest rate of 10%.

Calculate X.

(A) 5400

(B) 5420

(C) 5440

(D) 5460

(E) 5480

24.

A 20-year loan of 20,000 may be repaid under the following two methods:

i) amortization method with equal annual payments at an annual effective rate of 6.5%

ii) sinking fund method in which the lender receives an annual effective rate of 8% and the sinking fund earns an annual effective rate of j

Both methods require a payment of X to be made at the end of each year for 20 years.

Calculate j.

(A) $j \le 6.5\%$

(B) $6.5\% < j \le 8.0\%$

(C) $8.0\% < j \le 10.0\%$

(D) $10.0\% < j \le 12.0\%$

(E) $j > 12.0\%$

25.

A perpetuity-immediate pays X per year. Brian receives the first n payments, Colleen receives the next n payments, and Jeff receives the remaining payments. Brian's share of the present value of the original perpetuity is 40%, and Jeff's share is K.

Calculate K.

(A) 24%

(B) 28%

(C) 32%

(D) 36%

(E) 40%

26.

Seth, Janice, and Lori each borrow 5000 for five years at a nominal interest rate of 12%, compounded semi-annually.

Seth has interest accumulated over the five years and pays all the interest and principal in a lump sum at the end of five years.

Janice pays interest at the end of every six-month period as it accrues and the principal at the end of five years.

Lori repays her loan with 10 level payments at the end of every six-month period.

Calculate the total amount of interest paid on all three loans.

(A) 8718

(B) 8728

(C) 8738

(D) 8748

(E) 8758

27.

Bruce and Robbie each open up new bank accounts at time 0. Bruce deposits 100 into his bank account, and Robbie deposits 50 into his. Each account earns the same annual effective interest rate.

The amount of interest earned in Bruce's account during the 11th year is equal to X. The amount of interest earned in Robbie's account during the 17th year is also equal to X.

Calculate X.

(A) 28.0

(B) 31.3

(C) 34.6

(D) 36.7

(E) 38.9

28.

Ron is repaying a loan with payments of 1 at the end of each year for n years. The amount of interest paid in period t plus the amount of principal repaid in period $t + 1$ equals X.

Calculate X.

(A) $1 + \frac{v^{n-t}}{i}$

(B) $1 + \frac{v^{n-t}}{d}$

(C) $1 + v^{n-t}i$

(D) $1 + v^{n-t}d$

(E) $1 + v^{n-t}$

29.

At an annual effective interest rate of i, $i > 0\%$, the present value of a perpetuity paying `10 at the end of each 3-year period, with the first payment at the end of year 3, is 32.

At the same annual effective rate of i, the present value of a perpetuity paying 1 at the end of each 4-month period, with first payment at the end of 4 months, is X.

Calculate X.

(A) 31.6

(B) 32.6

(C) 33.6

(D) 34.6

(E) 35.6

30.

As of 12/31/03, an insurance company has a known obligation to pay $1,000,000 on 12/31/2007. To fund this liability, the company immediately purchases 4-year 5% annual coupon bonds totaling $822,703 of par value. The company anticipates reinvestment interest rates to remain constant at 5% through 12/31/07. The maturity value of the bond equals the par value.

Under the following reinvestment interest rate movement scenarios effective 1/1/2004, what best describes the insurance company's profit or (loss) as of 12/31/2007 after the liability is paid?

	Interest Rates Drop by ½%	Interest Rates Increase by ½%
(A)	+6,606	+11,147
(B)	(14,757)	+14,418
(C)	(18,911)	+19,185
(D)	(1,313)	+1,323

(E)	Breakeven	Breakeven

31.

An insurance company has an obligation to pay the medical costs for a claimant. Average annual claims costs today are $5,000, and medical inflation is expected to be 7% per year. The claimant is expected to live an additional 20 years.

Claim payments are made at yearly intervals, with the first claim payment to be made one year from today.

Find the present value of the obligation if the annual interest rate is 5%.

(A) 87,932

(B) 102,514

(C) 114,611

(D) 122,634

(E) Cannot be determined

32.

An investor pays $100,000 today for a 4-year investment that returns cash flows of $60,000 at the end of each of years 3 and 4. The cash flows can be reinvested at 4.0% per annum effective.

If the rate of interest at which the investment is to be valued is 5.0%, what is the net present value of this investment today?

(A) -1398

(B) -699

(C) 699

(D) 1398

(E) 2,629

33.

You are given the following information with respect to a bond:

par amount: 1000

term to maturity 3 years

annual coupon rate 6% payable annually

Term	Annual Spot Interest Rates
1	7%
2	8%
3	9%

Calculate the value of the bond.

(A) 906

(B) 926

(C) 930

(D) 950

(E) 1000

34.

You are given the following information with respect to a bond:

par amount: 1000

term to maturity 3 years

annual coupon rate 6% payable annually

Term	Annual Spot Interest Rates
1	7%
2	8%
3	9%

Calculate the annual effective yield rate for the bond if the bond is sold at a price equal to its value.

(A) 8.1%

(B) 8.3%

(C) 8.5%

(D) 8.7%

(E) 8.9%

35.

The current price of an annual coupon bond is 100. The derivative of the price of the bond with respect to the yield to maturity is -700. The yield to maturity is an annual effective rate of 8%.

Calculate the duration of the bond.

(A) 7.00

(B) 7.49

(C) 7.56

(D) 7.69

(E) 8.00

36.

Calculate the duration of a common stock that pays dividends at the end of each year into perpetuity. Assume that the dividend is constant, and that the effective rate of interest is 10%.

(A) 7

(B) 9

(C) 11

(D) 19

(E) 27

37.

Calculate the duration of a common stock that pays dividends at the end of each year into perpetuity. Assume that the dividend increases by 2% each year and that the effective rate of interest is 5%.

(A) 27

(B) 35

(C) 44

(D) 52

(E) 58

38. – 44. skipped

45.

You are given the following information about an investment account:

Date	Value Immediately Before Deposit	Deposit
January 1	10	
July 1	12	X
December 31	X	

Over the year, the time-weighted return is 0%, and the dollar-weighted (money-weighted) return is *Y*.

Calculate *Y*.

(A) -25%

(B) -10%

(C) 0%

(D) 10%

(E) 25%

46.

Seth borrows X for four years at an annual effective interest rate of 8%, to be repaid with equal payments at the end of each year. The outstanding loan balance at the end of the third year is 559.12.

Calculate the principal repaid in the first payment.

(A) 444

(B) 454

(C) 464

(D) 474

(E) 484

47.

Bill buys a 10-year 1000 par value 6% bond with semi-annual coupons. The price assumes a nominal yield of 6%, compounded semi-annually.

As Bill receives each coupon payment, he immediately puts the money into an account earning interest at an annual effective rate of *i*.

At the end of 10 years, immediately after Bill receives the final coupon payment and the redemption value of the bond, Bill has earned an annual effective yield of 7% on his investment in the bond.

Calculate *i*.

(A) 9.50%

(B) 9.75%

(C) 10.00%

(D) 10.25%

(E) 10.50%

48.

A man turns 40 today and wishes to provide supplemental retirement income of 3000 at the beginning of each month starting on his 65th birthday. Starting today, he makes monthly contributions of *X* to a fund for 25 years. The fund earns a nominal rate of 8% compounded monthly.

On his 65th birthday, each 1000 of the fund will provide 9.65 of income at the beginning of each month starting immediately and continuing as long as he survives.

Calculate *X*.

(A) 324.73

(B) 326.89

(C) 328.12

(D) 355.45

(E) 450.65

49.

Happy and financially astute parents decide at the birth of their daughter that they will need to provide 50,000 at each of their daughter's 18th, 19th, 20th and 21st birthdays to fund her college education. They plan to contribute X at each of their daughter's 1st through 17th birthdays to fund the four 50,000 withdrawals. If they anticipate earning a constant 5% annual effective rate on their contributions, which the following equations of value can be used to determine X, assuming compound interest?

(A) $X[v_{.05}^{1} + v_{.05}^{2} + ...v_{.05}^{17}] = 50{,}000[v_{.05}^{1} + ...v_{.05}^{4}]$

(B) $X[(1.05)^{16} + (1.05)^{15} + ...(1.05)^{1}] = 50{,}000[1 + ...v_{.05}^{3}]$

(C) $X[(1.05)^{17} + (1.05)^{16} + ...1] = 50{,}000[1 + ...v_{.05}^{3}]$

(D) $X[(1.05)^{17} + (1.05)^{16} + ...(1.05)^{1}] = 50{,}000[1 + ...v_{.05}^{3}]$

(E) $X[(1 + v_{.05}^{1} + ...v_{.05}^{17}] = 50{,}000[v_{.05}^{18} + ...v_{.05}^{22}]$

50.

A 1000 bond with semi-annual coupons at $i^{(2)}$ = 6% matures at par on October 15, 2020.

The bond is purchased on June 28, 2005 to yield the investor $i^{(2)}$ = 7%. What is the purchase price?

Assume simple interest between bond coupon dates and note that:

Date	Day of the Year
April 15	105
June 28	179
October 15	288

(A) 906

(B) 907

(C) 908

(D) 919

(E) 925

The following information applies to questions 51 thru 53.

Joe must pay liabilities of 1,000 due 6 months from now and another 1,000 due one year from now. There are two available investments:

a 6-month bond with face amount of 1,000, a 8% nominal annual coupon rate convertible semiannually, and a 6% nominal annual yield rate convertible semiannually; and

a one-year bond with face amount of 1,000, a 5% nominal annual coupon rate convertible semiannually, and a 7% nominal annual yield rate convertible semiannually

51.

How much of each bond should Joe purchase in order to exactly (absolutely) match the liabilities?

	Bond I	Bond II
(A)	1	.97561
(B)	.93809	1
(C)	.97561	.94293
(D)	.93809	.97561
(E)	.98345	.97561

The following information applies to questions 51 thru 53.

Joe must pay liabilities of 1,000 due 6 months from now and another 1,000 due one year from now. There are two available investments:

a 6-month bond with face amount of 1,000, a 8% nominal annual coupon rate convertible semiannually, and a 6% nominal annual yield rate convertible semiannually; and

a one-year bond with face amount of 1,000, a 5% nominal annual coupon rate convertible semiannually, and a 7% nominal annual yield rate convertible semiannually

52.

What is Joe's total cost of purchasing the bonds required to exactly (absolutely) match the liabilities?

(A) 1894

(B) 1904

(C) 1914

(D) 1924

(E) 1934

The following information applies to questions 51 thru 53.

Joe must pay liabilities of 1,000 due 6 months from now and another 1,000 due one year from now. There are two available investments:

a 6-month bond with face amount of 1,000, a 8% nominal annual coupon rate convertible semiannually, and a 6% nominal annual yield rate convertible semiannually; and

a one-year bond with face amount of 1,000, a 5% nominal annual coupon rate convertible semiannually, and a 7% nominal annual yield rate convertible semiannually

53.

What is the annual effective yield rate for investment in the bonds required to exactly (absolutely) match the liabilities?

(A) 6.5%

(B) 6.6%

(C) 6.7%

(D) 6.8%

(E) 6.9%

54.

Matt purchased a 20-year par value bond with semiannual coupons at a nominal annual rate of 8% convertible semiannually at a price of 1722.25. The bond can be called at par value X on any coupon date starting at the end of year 15 after the coupon is paid. The price guarantees that Matt will receive a nominal annual rate of interest convertible semiannually of at least 6%.

Calculate X.

(A) 1400

(B) 1420

(C) 1440

(D) 1460

(E) 1480

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55.

Toby purchased a 20-year par value bond with semiannual coupons at a nominal annual rate of 8% convertible semiannually at a price of 1722.25. The bond can be called at par value 1100 on any coupon date starting at the end of year 15.

What is the minimum yield that Toby could receive, expressed as a nominal annual rate of interest convertible semiannually?

(A) 3.2%

(B) 3.3%

(C) 3.4%

(D) 3.5%

(E) 3.6%

56.

Sue purchased a 10-year par value bond with semiannual coupons at a nominal annual rate of 4% convertible semiannually at a price of 1021.50. The bond can be called at par value X on any coupon date starting at the end of year 5. The price guarantees that Sue will receive a nominal annual rate of interest convertible semiannually of at least 6%.

Calculate X.

(A) 1120

(B) 1140

(C) 1160

(D) 1180

(E) 1200

57.

Mary purchased a 10-year par value bond with semiannual coupons at a nominal annual rate of 4% convertible semiannually at a price of 1021.50. The bond can be called at par value 1100 on any coupon date starting at the end of year 5.

What is the minimum yield that Mary could receive, expressed as a nominal annual rate of interest convertible semiannually?

(A) 4.8%

(B) 4.9%

(C) 5.0%

(D) 5.1%

(E) 5.2%